



R20.00005 — March 18, 2021 9:12 am CDT

Field-induced freezing in the unfrustrated Ising antiferromagnet

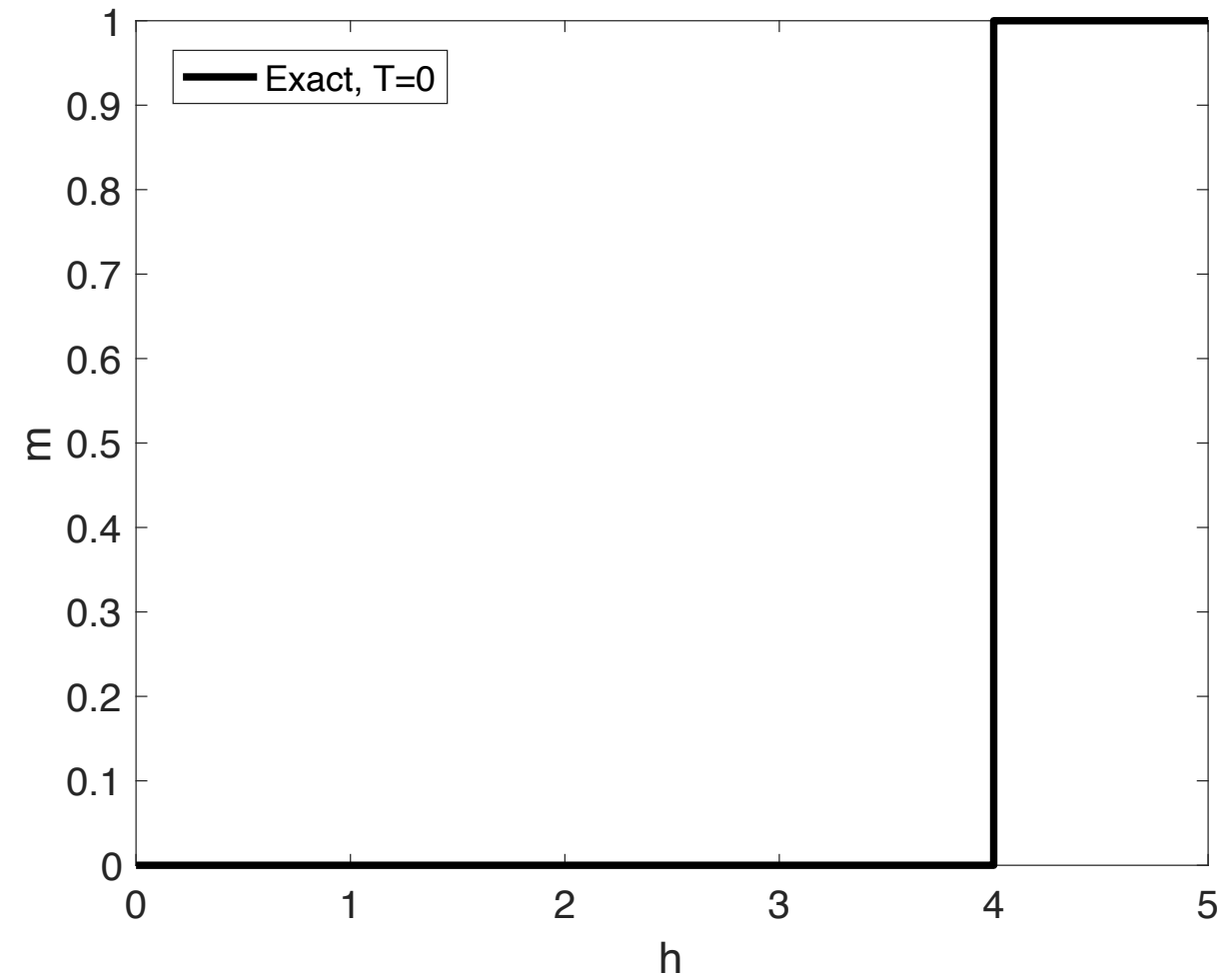
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placed at the
US Department of Energy

Better title: *Stable frozen states without disorder*

2D Ising AFM

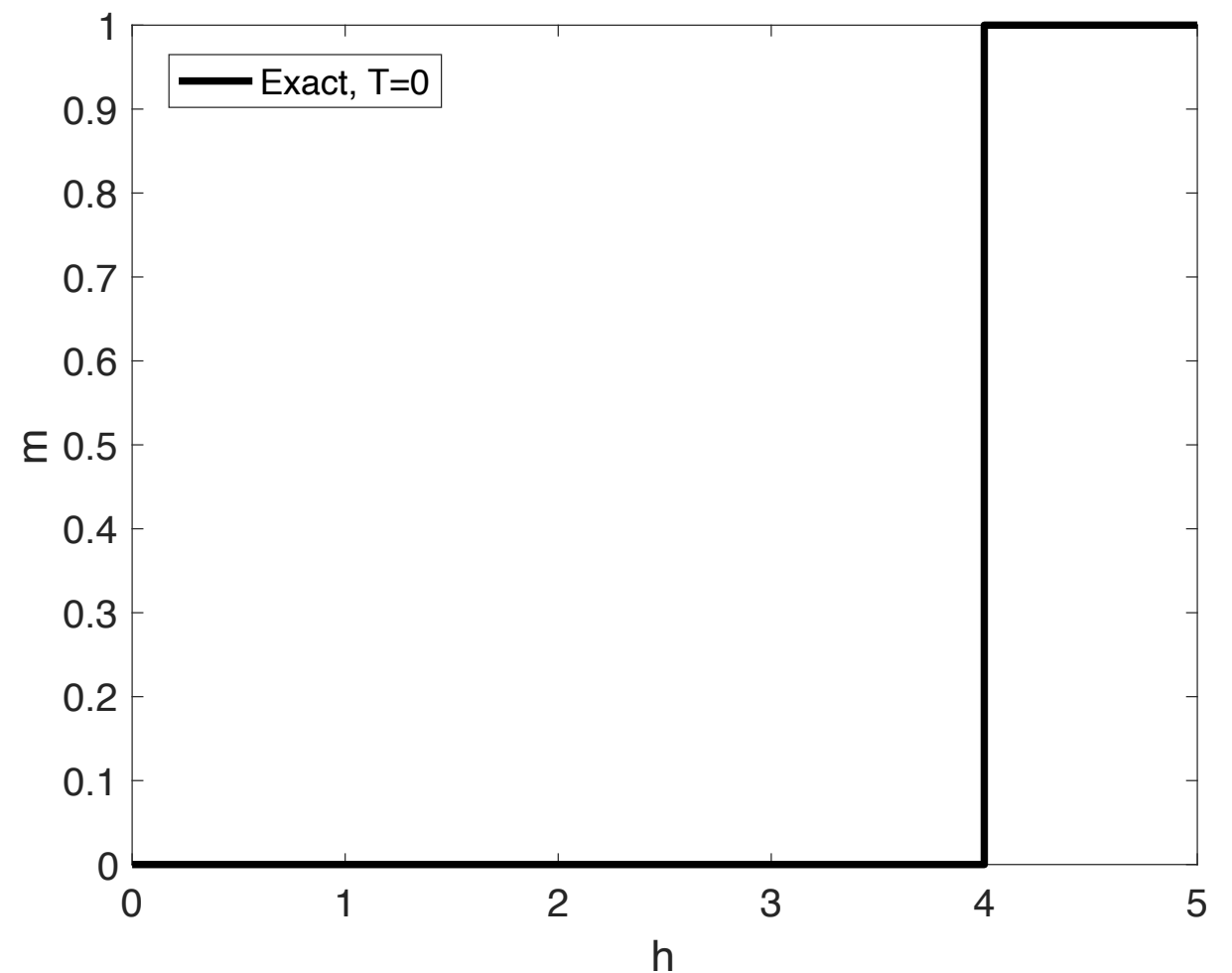
- ❖ $H = J \sum_{\langle i,j \rangle} \sigma_i \sigma_j - h \sum_i \sigma_i$
- ❖ Square lattice w / PBC
- ❖ Antiferromagnet
 - ❖ 2-fold degenerate GS
 - ❖ $T_c = 2.26\dots$
 - ❖ Simplest model with PT
- ❖ With field: poorly studied



$$m(T = 0, h) = \begin{cases} 0 & h < 4 \\ 1 & h > 4 \end{cases}$$

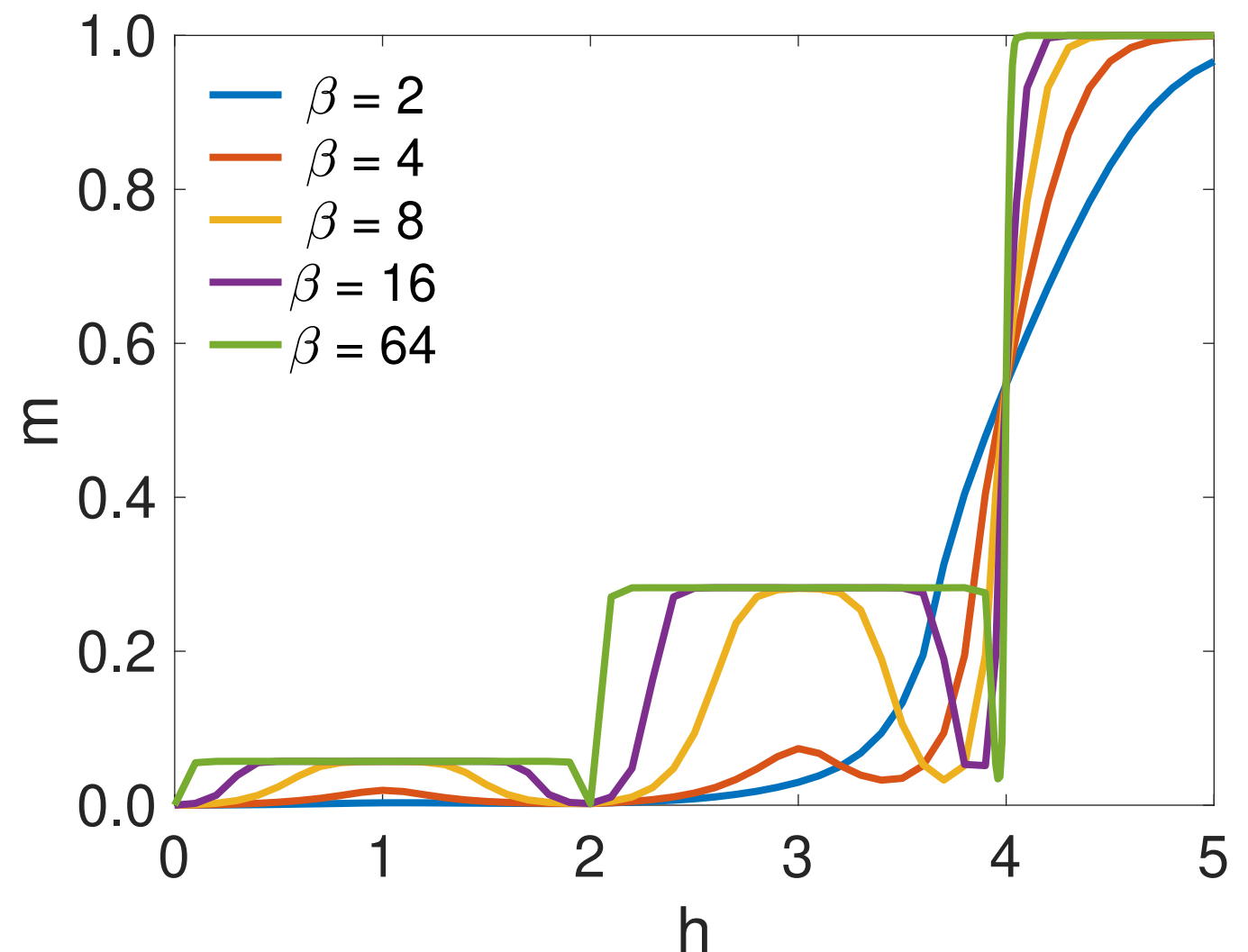
Dynamics

- ❖ Metropolis Monte Carlo
- ❖ Single-spin-flip updates
 - ❖ Choose spin at random
 - ❖ Flip with probability $P = \min [1, e^{-\Delta E/T}]$
- ❖ Quench:
 - ❖ Start from $T = \infty$ (totally random) state
 - ❖ Instant quench to T
- ❖ What happens?



Actual behavior

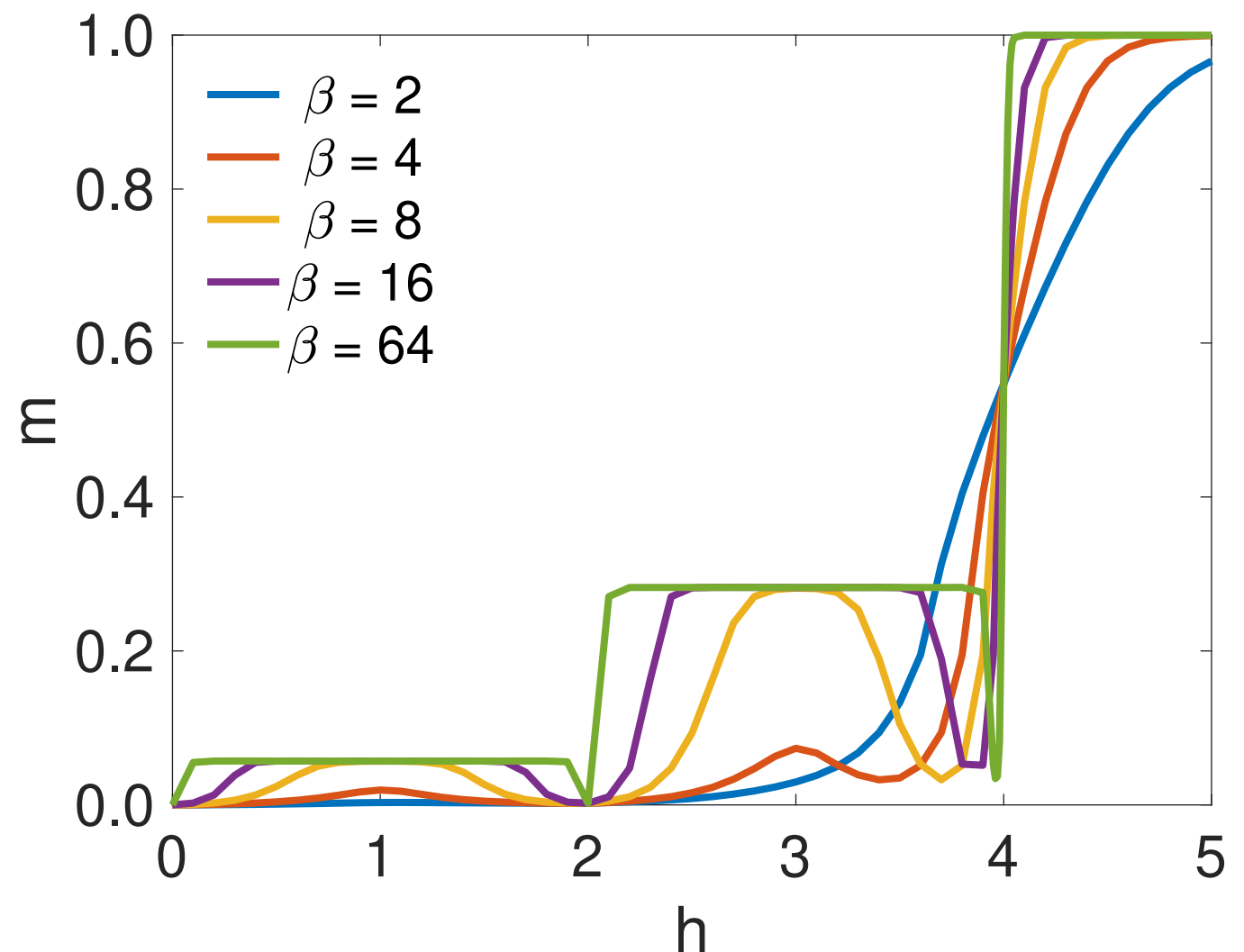
- ❖ Instantaneous quench to finite $T < T_c$
- ❖ High T : EQ
- ❖ Low T : Non-ergodic
 - ❖ Plateaus
 - ❖ Stable frozen states
 - ❖ No intrinsic disorder
- ❖ Valleys of ergodicity
- ❖ Boundary / size unimportant



Zero temperature magnetization

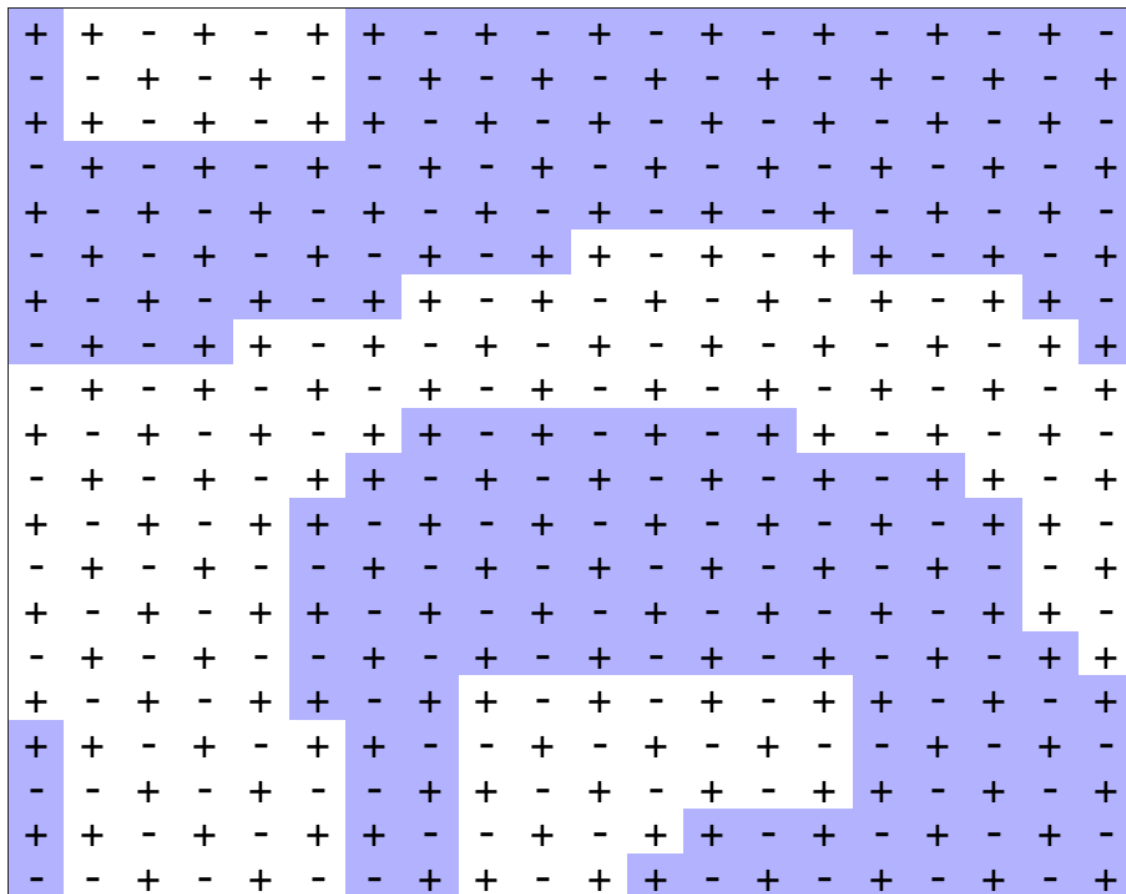
$$m(T = 0, h) = \begin{cases} 0 & h = 0, \\ 0.057 & 0 < h < 2J, \\ 0 & h = 2J, \\ 0.282 & 2J < h < 4J, \\ 0.55 & h = h_s = 4J, \\ 1 & h > h_s. \end{cases}$$

- ❖ Ergodic for $h = 0, \pm 2, \pm 4$
- ❖ From now on: $T = 0$

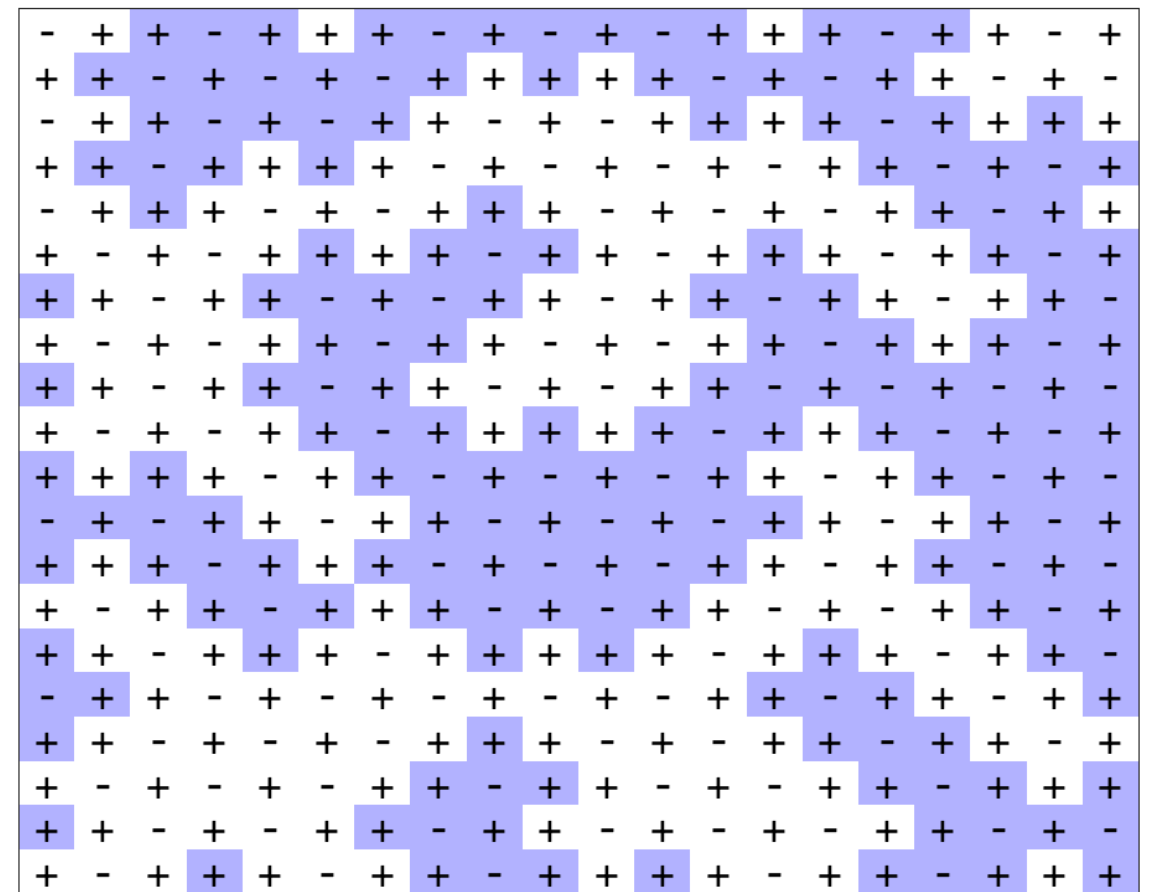


What is happening?

First plateau: $h = 1$



Second plateau: $h = 3$



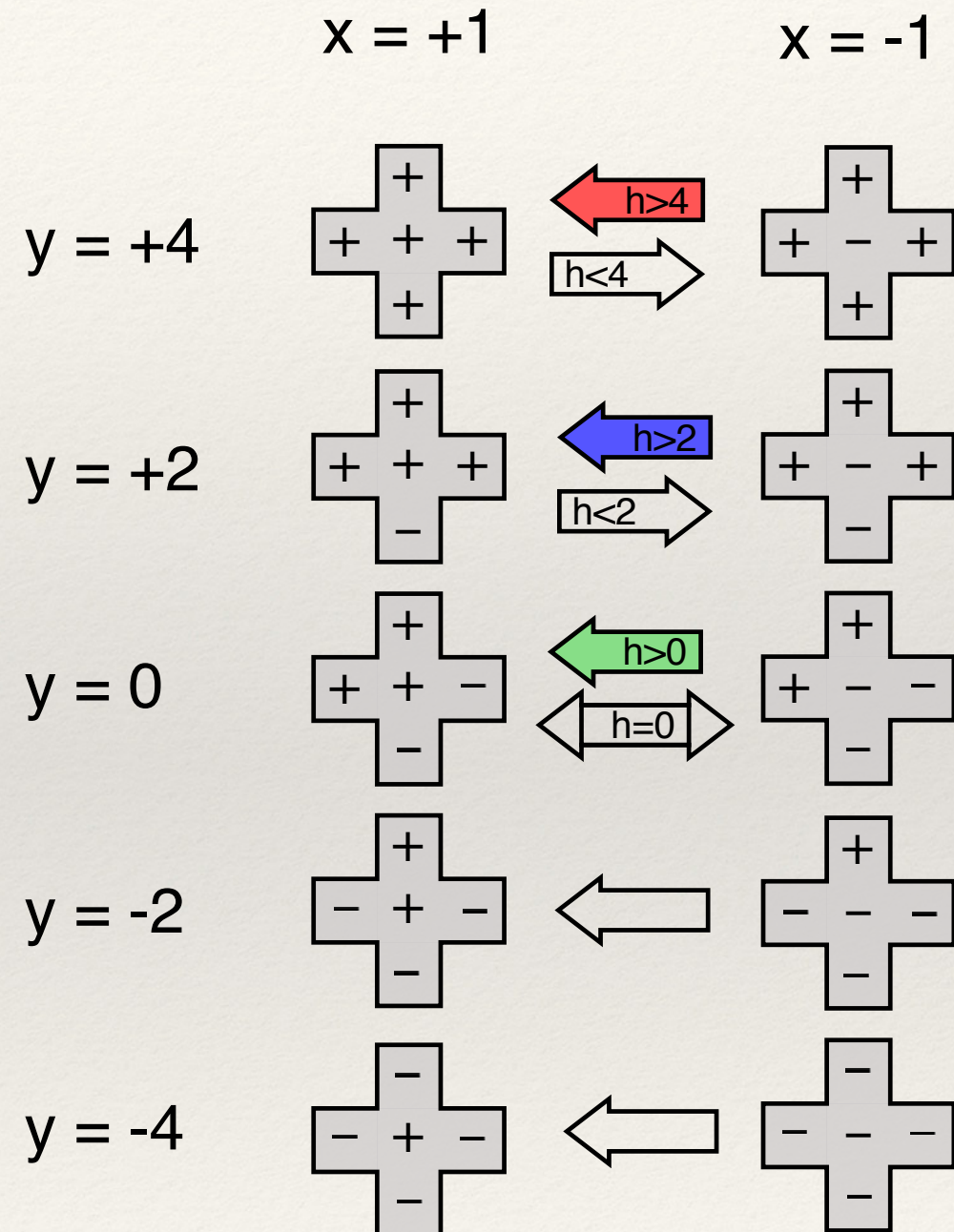
Freezing Mechanism

$$x = \sigma_i = \pm 1$$

$$y = \sum_j \sigma_j = 0, \pm 2, \pm 4$$

$$P = \min \left[1, e^{-(y-h)\Delta x/T} \right]$$

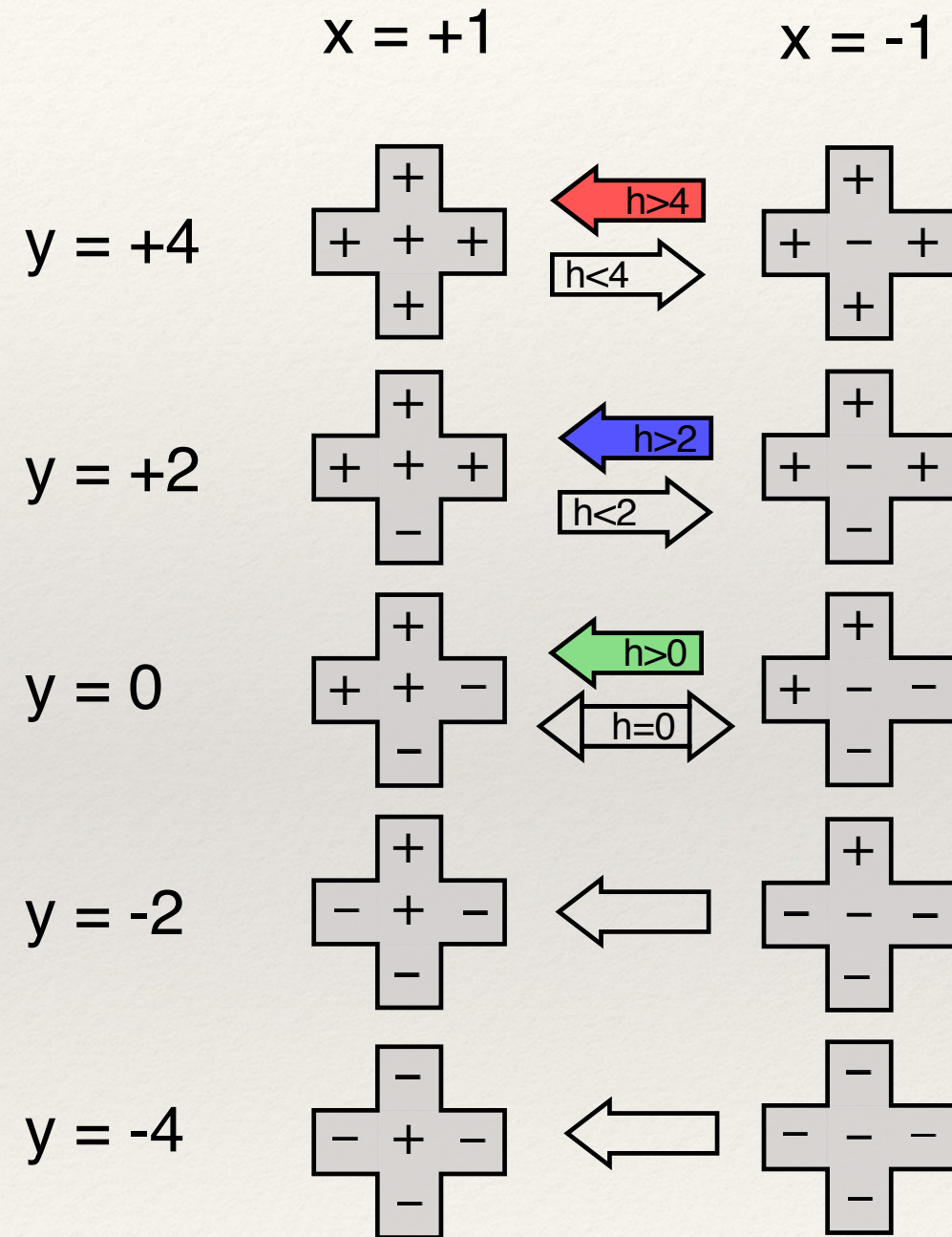
- ❖ 10 local spin configurations
- ❖ 5 pairs



Zero temperature dynamics

$$\Delta E = (y - h)\Delta x$$

- ❖ $T = 0$ dynamics:
 - ❖ Accept if $\Delta E \leq 0$
 - ❖ Reject if $\Delta E > 0$
- ❖ $\Delta E = 0$: *reversible* update
- ❖ Reversible updates when $h = y = 0, \pm 2, \pm 4$
 - ❖ Valleys of ergodicity



$$h = 0$$

- ❖ Maps onto ferromagnet
- ❖ Bulk domains and straight domain walls stable
- ❖ $T = 0$ quench, stuck in stripe state $w/P = 0.3390\dots$
 - ❖ Connection to critical percolation theory
- ❖ Otherwise reach G.S.

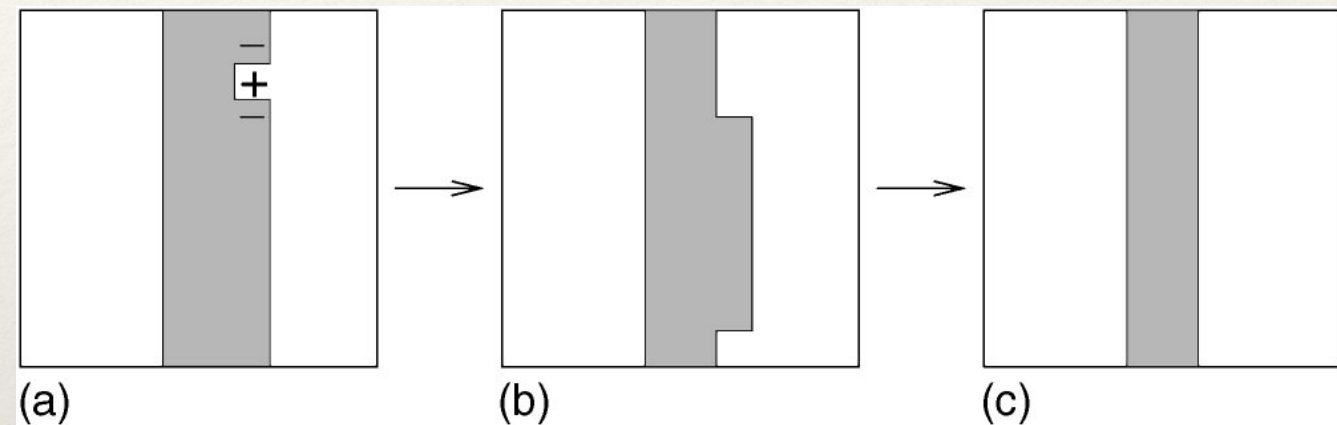
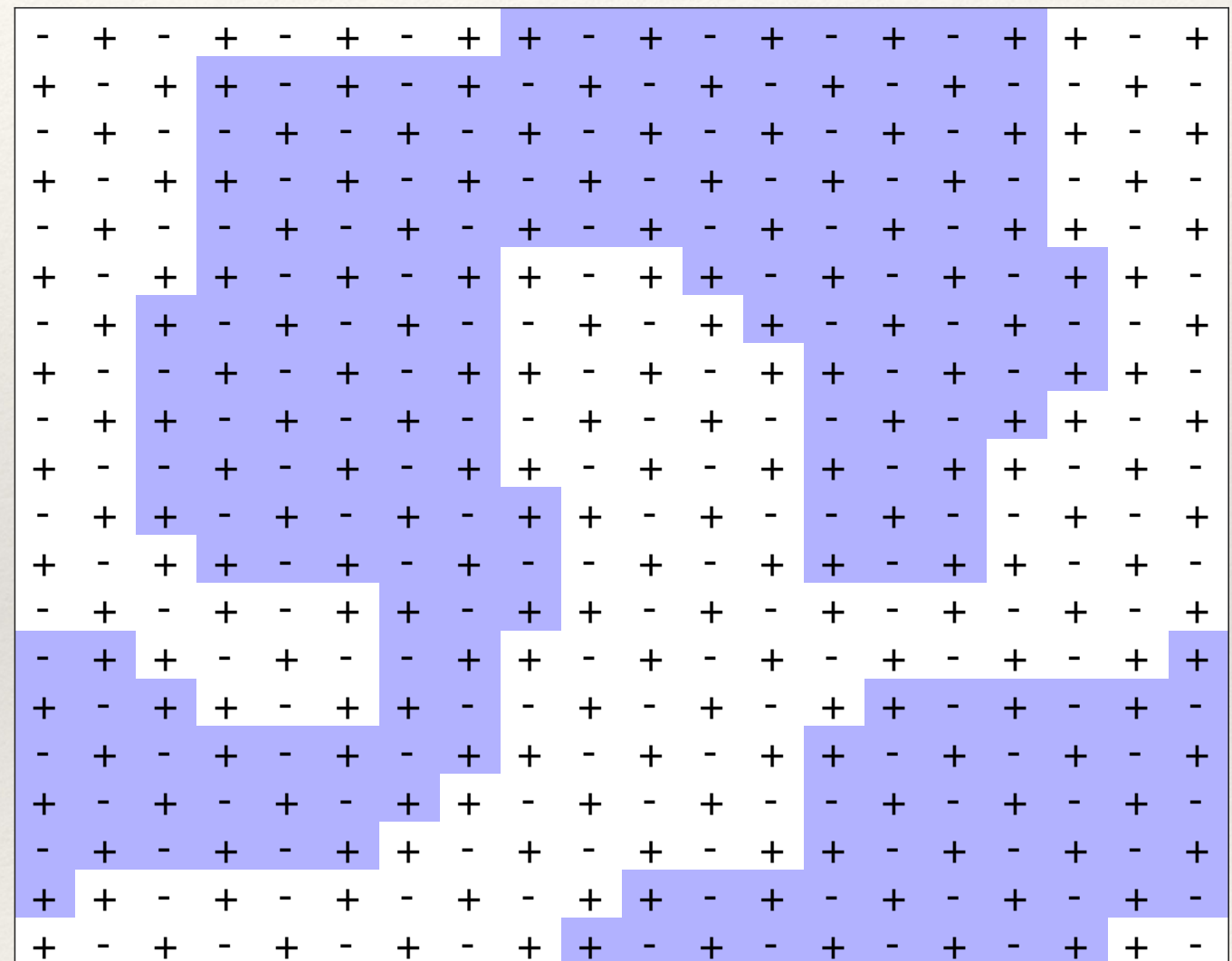


FIG. 13. Relaxation of a stripe state in two dimensions at small nonzero temperature: (a) nucleation of a dent (freely flippable spins are indicated); (b) diffusive growth of the dent; (c) dent reaches the system size and hence the domain wall steps to the left. This overall process ultimately leads to the disappearance of the stripe.

Spirin, Krapivsky, Redner, PRE **65** 016119 (2001)

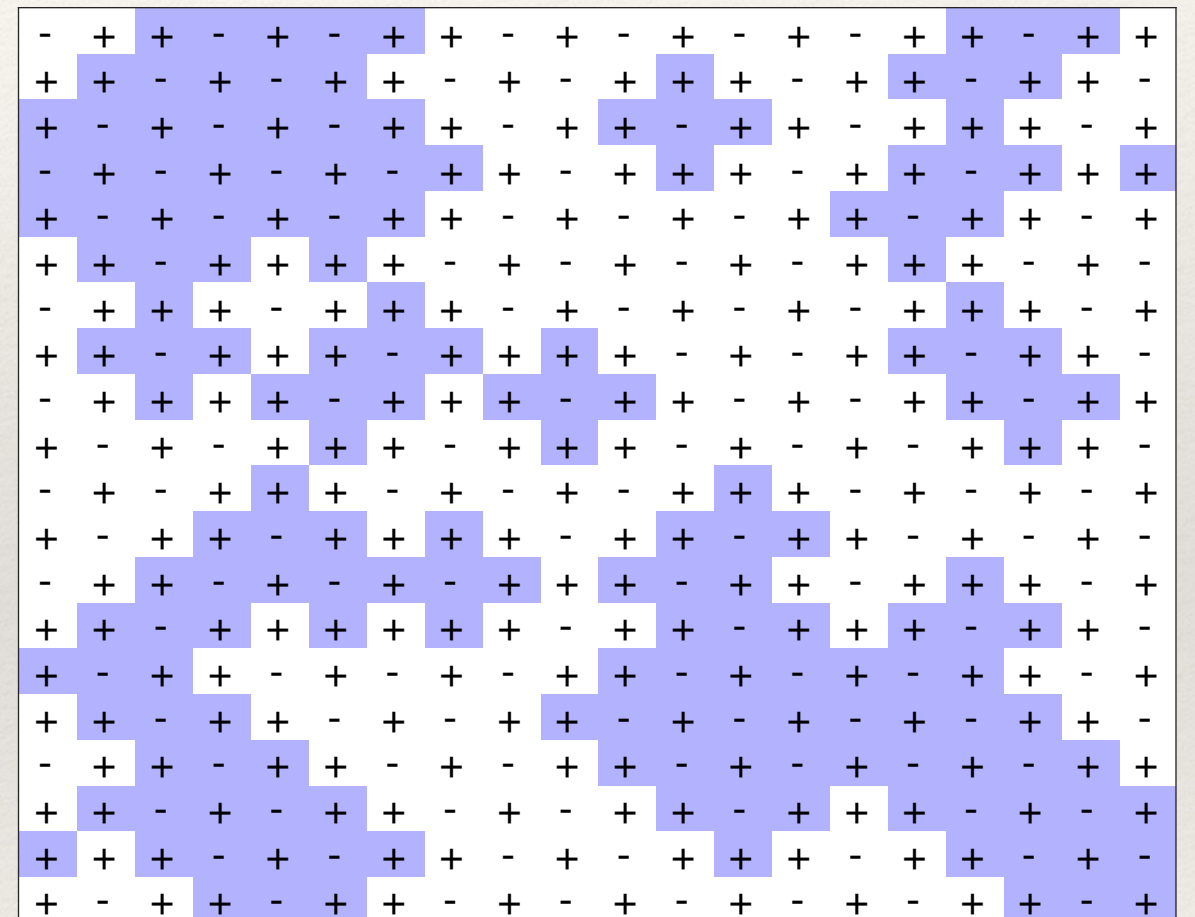
First plateau

- ❖ $0 < h < 2J$
- ❖ Corner domain walls now stable
- ❖ Corners host excess + spin
- ❖ Net magnetization 0.057



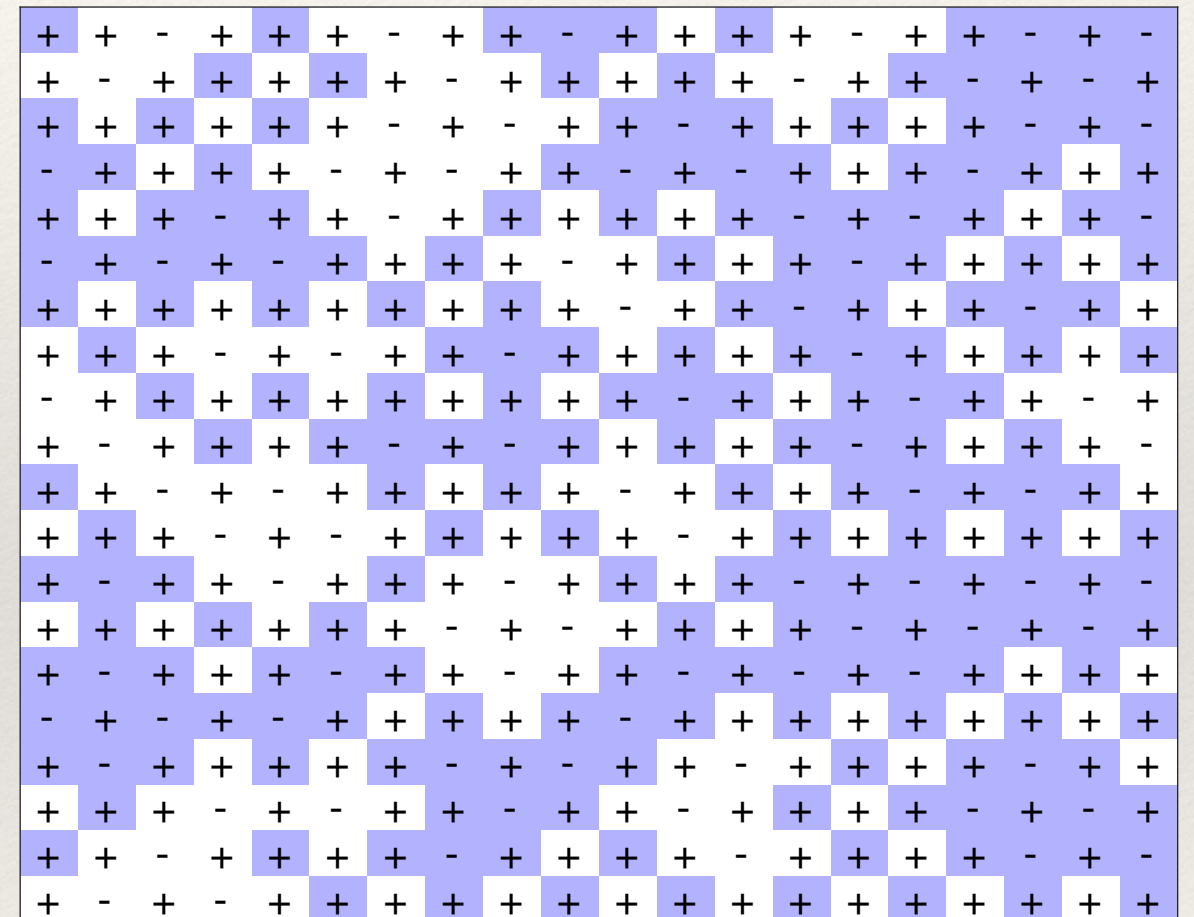
Second plateau

- ❖ Straight line DW not stable
- ❖ Corners / diagonal DW stable
- ❖ Excess + spin along DW
- ❖ Net magnetization 0.282



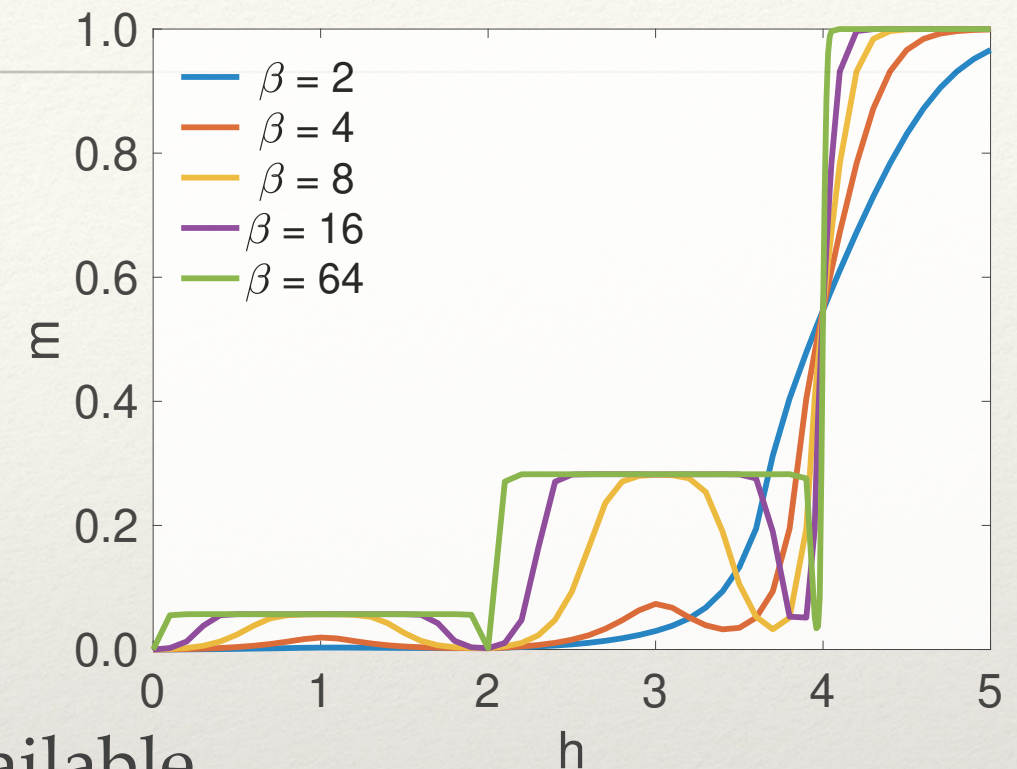
$$h = h_s$$

- ❖ Does not freeze
- ❖ Vanishing T dependence
- ❖ Coexistence of
 - ❖ 2x AFM G.S.
 - ❖ Fully-polarized state



Summary

- ❖ Quenches in Ising AFM + field
 - ❖ Stable plateaus w/o disorder
 - ❖ In plateau: all spins unflippable
 - ❖ Stable local configurations
- ❖ Ergodicity restored when $\Delta E = 0$ updates available
 - ❖ Useful for understanding when MC fails
- ❖ Monte Carlo dynamics $\neq$ physical dynamics
 - ❖ We choose the dynamics
 - ❖ Plateau states are local energy minima under these dynamics
 - ❖ Choosing dynamics rearranges energy surface



Why should we care?

- ❖ MCMC may be the most widely used single algorithm
 - ❖ Understanding non-ergodicity key to understanding failures
- ❖ Finding GS is an **optimization** problem in high-dim. space
 - ❖ Spin glass, protein folding, machine learning
- ❖ Optimize some function: $f(x_1, x_2, x_3 \dots)$
 - ❖ But we choose the dynamics, can be nonlocal w.r.t. $\vec{x}$
 - ❖ Nonlocal moves can destroy local minima
- ❖ Sticky / glassy problems usually slow
 - ❖ This case is FAST
 - ❖ Meaningful results on a **laptop**

Thanks for your attention!

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For paper, source code

iaizzi.me/aps2021/

or scan:



Paper:

A. Iaizzi, *Phys. Rev. E* **102** 032112 (2020)

[arXiv:2001.09268](https://arxiv.org/abs/2001.09268)

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